5.2.2

$\Rightarrow$ $A(\theta)_{i_1\cdots i_s} = A(\theta)\left(\frac{\partial}{\partial x^{i_1}} \cdots \frac{\partial}{\partial x^{i_s}}\right) = \frac{1}{s!}\sum_{\pi}(-1)^{\pi}\theta\left(\frac{\partial}{\partial x^{\pi(i_1)}} \cdots \frac{\partial}{\partial x^{\pi(i_s)}}\right) = \frac{1}{s!}\sum_{\pi}(-1)^{\pi}\theta_{\pi(i_1)\cdots\pi(i_s)}$

$\Leftarrow$ $A(\theta)(X_1\cdots X_s) = \sum_{j_1\cdots j_s=1}^{n} a_{1j_1}\cdots a_{sj_s} A(\theta)_{j_1\cdots j_s} = \frac{1}{s!}\sum_{\pi}(-1)^{\pi}\sum_{\pi(j_1)\cdots\pi(j_s)=1}^{n} a_{1\pi(j_1)}\cdots a_{s\pi(j_s)}\theta_{\pi(j_1)\cdots\pi(j_s)}$ = 右

5.2.8

(1) $\Rightarrow$ 显然

$\Leftarrow$ $\forall X_1\cdots X_n \in T_p(M)$ $\quad \omega(X_1\cdots X_n) = \sum_{i_1\cdots i_n=1}^{n} \alpha_{i_1}\cdots\alpha_{i_n}\omega(e_1\cdots e_n) = 0 \Rightarrow \omega = 0$

(2) $\omega(X_1\cdots X_n) = \sum_{i_1\cdots i_n=1}^{n} a_{1i_1}\cdots a_{ni_n}(-1)^{\pi(i_1\cdots i_n)}\omega(e_1\cdots e_n) = \det(a_j^i)\,\omega(e_1\cdots e_n)$

(3) $\omega\wedge 0(X_1\cdots X_{r+s}) = \frac{1}{r!s!}\sum_{\pi}(-1)^{\pi}\omega(X_{\pi(1)}\cdots X_{\pi(r)})\cdot 0(X_{\pi(r+1)}\cdots X_{\pi(r+s)}) = 0$

$0\wedge\omega(X_1\cdots X_{r+s}) = 0$ (同理)

(4) $\Rightarrow$ $(\omega_1\wedge\omega_2)(X_1, X_2) = \sum_{\pi}(-1)^{\pi}\omega_1(X_{\pi(1)})\omega_2(X_{\pi(2)})$

$= -\sum_{\pi}(-1)^{\pi}\omega_2(X_{\pi(2)})\omega_1(X_{\pi(1)})$

$= -(\omega_2\wedge\omega_1)(X_1, X_2)$

$\Leftarrow$ $\omega\wedge\omega = -\omega\wedge\omega$

5.2.9.

(1) $d(\omega+\eta) = 0$ + $d(\lambda\omega) = 0$ + $d(\omega\wedge\eta) = 0$

(2) $\underline{\sum B_D{}^s(M) \subset \sum Z_D{}^s(M)}$

$\forall\ \omega_1\ \omega_2 \in \sum B_D{}^s(M)$ $\ \exists\ \eta_1, \eta_2$ s.t. $\omega_1 = d(\eta_1)$ $\quad \omega_2 = d(\eta_2)$

$\omega_1 + \omega_2 = d(\eta_1) + d(\eta_2) = d(\eta_1 + \eta_2)$

$\lambda\cdot\omega_1 = \lambda\cdot d(\eta_1) = d(\lambda\eta_1)$

$\omega_1\wedge\omega_2 = d(\eta_1)\wedge d(\eta_2) = d(\eta_1\wedge\eta_2)$

2.12

(1) $F^*\omega = \sum_{i=1}^{n} a_i\frac{\partial}{\partial x^i}(F) = 0$

(2) $(G\circ F)^*\omega(X_1\cdots X_s) = \omega(F_*\circ G_* X_1\cdots F_*\circ G_* X_s) = F^*\omega(G_* X_1\cdots G_* X_s) = F^*\circ G^*\omega(X_1\cdots$

(3) 同上